Note: The research proposals do not have to be one of the titles provided in this List, but it is recommended that the research is supported by experts available within the faculty of the Nazarbayev University School of Engineering and School of Science and Technology. Candidates may wish to generate their own titles. We also encourage the PhD candidates to contact potential Faculty Members for additional information on the following PhD topics.

Prof. Dongming Wei dongming.wei@nu.edu.kz


The goals of the project is to provide accurate and efficient device structural models with efficient numerical simulation and mathematical analysis aiming to help design and optimize structural components in the modern pressure and strain sensors made of the modern nonlinear materials with applications in MEMS and NAMS (The project is funded by an ORAU grant, with engineering co-PI’s: Dichuan Zhang and Match Wai Long Ko et al.).

2. Modeling Extrusion of non-Newtonian fluids for Applications of Polymer and Functional Ceramic Materials

The goals of the project is to provide accurate and efficient mathematical models and the associated simulations for extrusion of polymer and functional ceramic materials in the extrusion die interiors. The goal is to help design and optimize the design of extrusion die heads for coding and thin film productions. This will involve solving the multi-physics equations of fluid-thermal-solid interactions analytically and numerically (the part for ceramic materials are jointly conducted with Boris Golman of NU SEng).

Modeling of MEMS/NEMS pull-in Devices

This research would be to incorporate nonlinear materials parameters and/or large deformations into the models of nonlinear materials such as graphene, polyimide, and many high strength metals subject to various loading conditions, including electrical loads for some basic structural elements in energy and bio-medical pull-in devices.
4. Modeling of Polymer Extrusion Die Flows

In Kazakhstan, a plant for producing 18 tons of thin films per year, as well as producing 48 million polypropylene bags per year is in operation since 2012. In addition, the integrated gas chemical complex in Atyrau has the capacity of producing polyethylene and polypropylene pellets as well as some plastic films. The goal of this project is to adapt and develop the polymer extrusion technology for the local industry and provide novel design of extrusion dies for the purpose of processing the polyethylene and polypropylene resins produced in Kazakhstan into plastic sheets and thin films, to create prototypes of these designed die, and to perform experimental validations of the designs. The project objectives are:

1) Collect samples and rheology data from local polymer producers and use experimental and statistical regression methods to identify the mathematical models and to estimate the rheological parameters for extrusion die design;

2) To study the feasibility of applying the current state of the art design simulation algorithms developed by the team members to the Kazakhstan polymer products for thin film production;

3) To build prototypes of extrusion dies from our design and to test the prototypes on Kazakhstan polymers with experimental validations;

4) Improve the current extrusion die design theory, algorithm and software, develop new and improved die designs for the local polymer products.

1. Dynamical and Statistical Complexity of Coupled Nonlinear Oscillators

We often encounter in many branches of Science and Engineering physical systems consisting of coupled nonlinear oscillators, which may be mechanical, molecular, electrical or optical. Each unit of these systems is well understood, and its behavior may be periodic or chaotic. However, if we connect them all together in certain ways, the system as a whole starts to display remarkable and often unexpected behaviors, where all units may oscillate in synchrony, or all will be asynchronous or half will be synchronized and the other half not! In this project we will analyze such global phenomena on a variety of models with applications to Physics (Optics and Solid State) and Biology (Neuroscience) to find out how they depend on local properties like connectivity and coupling parameters between the units.

2. Invariant Manifolds of 4-dimensional Maps and their Applications to Solid State Physics and Accelerator Dynamics

Four – Dimensional Maps (or discrete dynamical systems) are very important because they are analytically and numerically more tractable than systems of nonlinear differential equations. In recent years it has been observed that some of their dynamical characteristics, such as invariant manifolds, can be described by convergent series expansions and thus yield very accurate initial conditions for analyzing various important physical phenomena. In this project we will show how to obtain such representations of invariant manifolds and use them to study energy localization in 1-dimensional molecular lattices, as well as estimate stability regions of proton beams in high energy accelerators, such as those used at CERN in Geneva.

Prof. Piotr Skrzypacz

1. Composite Finite Elements for Non-Newtonian flows

Most often non-linear PDEs or systems of coupled non-linear PDEs are used to model time dependent complex phenomena in engineering, like flows of non-Newtonian fluids, temperature distribution in chemical reactors. In practice, one would like to control the processes modeled by such non-linear PDEs. In our project we plan to develop new composite finite element solvers
for non-linear boundary value problems in order to optimize industrial processes like ceramics extrusion in 3D printing [1,2]. The composite elements [3] are promising for convection-dominated problems when using Local Projection Stabilization (LPS). The main work will focus on finite element analysis for non-Newtonian flows. The in-house MATLAB finite element code will be extended in order to simulate several real-world scenarios.


2. Composite Finite Element Methods for problems in chemical reaction engineering

The government of the Republic of Kazakhstan has defined the development of chemical and petrochemical industries as one of top priorities for the country. Recently, Kazakhstan has launched new plants for manufacturing chemical and petrochemical products with high added value. The core of this industrial development is based on engineering of chemical reactors, such as fixed-bed and fluidized-bed catalytic reactors. Since reactions occurring in industrial plants are very frequently exothermic, the precise control of chemical reactors is very important to avoid formation of hot spots which can cause the catalyst sintering, reactor malfunction and even lead to industrial accidents. This project will be an interdisciplinary research between Applied Mathematics and Engineering. Mathematics part includes finite element and optimization techniques. Composite Finite Element Method (FEM) [1] is a way to find approximate solutions to the Boundary Value Problems associated with the chemical reaction to be considered.

In practice, one would like to control the processes modeled by such non-linear PDEs. This requires not only fast solvers for PDEs but also sophisticated approaches in order to solve optimization problems with PDEs as constraints. In this project we want to focus on Dirichlet boundary control problems [2,3] in chemical reaction engineering, which model control of temperature in chemical catalytic reactors by prescribing an optimal temperature on the boundary in order to avoid hot-spots. In our project we plan to develop new finite element solvers that can also be used for solution of boundary control problems in various industrial processes.
Convergence Analysis of Retarded Fuzzy Neural Networks

The objective of this research is to address some fundamental stability and convergence analysis problems that arise in fuzzy neural networks. In particular, we will consider stability analysis of periodic and almost periodic solutions of retarded fuzzy shunting inhibitory neural networks (SICNNs). Introduced by Bouzerdoum and Pinter, SICNNs are modeled by the nonlinear ordinary differential equation. Due to its great applicability to various areas such as image processing, adaptive pattern recognition, speech and perception SICNNs has become one of the popular subjects in recent years. For instance, SICNNs applied color images improve sharpness and color constancy of images. Moreover, Bermak and Bouzerdoum implemented SICNNs to a real-time mixed analog digital VLSI and attained remarkable speed up in processing.

Asymptotic behavior of SIR and SEIS epidemic models with piecewise constant argument of generalized type

This research aims at making substantial contribution to develop and analyze compartmental models in epidemiology. The main research efforts will focus on the understanding complex behavior of SIR and SEIS epidemic models that functionally depend on the piecewise constant
arguments. We aim at studying local and global stability properties of disease free equilibrium to these models. Furthermore, we aim to investigate existence of periodic solutions for seasonally forced SIR and SEIS with piecewise constant arguments of generalized type.


Yogi Erlangga (Mathematics, SST), yogi.erlangga@nu.edu.kz

Co-adviser: Alejandro Castro-Castilla (Mathematics, SST)

**Title: Sparsity Promoting Inverse Problem in Seismic Imaging**

Given a mathematical model of some physical phenomena that depend on some parameters, we are interested in the valuation of parameters that lead to certain observed data. This data is usually obtained via a series of controlled experiments under various inputs. In oil prospecting, geophysicists send acoustic or elastic waves and record waves reflected by the Earth’s subsurface. Different subsurface structures lead to different responses, and the task now is to find a particular structure which produces data that best fits the recorded one. The resulting structure can then be evaluated for the prospect of locating, e.g., oil reserve. This so-called inverse problem can also be formulated as a PDE-constrained optimization.

Seismic imaging based on the above optimization procedure, known as Full-Waveform Inversion (FWI), starts to gain popularity in the 1990s, and is demonstrated to be an effective technique for reflection data with long offset. The above procedure however involves big data and requires computations of the solution of the partial differential equations (PDEs), which model the underlying physics of wave propagations in either acoustic or elastic media. In the frequency domain, for the latter, the immediate challenge is in the accurate approximation of PDEs and the efficient solve of the large algebraic system.

The main objectives of this project are
1. fast and efficient numerical solution for large-scale PDE-constrained optimization, which includes capability to solve multiple problems, associated with multiple shots. The numerical method may consists of the PDE solver, which include higher-order numerical approximations and fast, memory-saving iterative solvers for sparse system, and the sparsity-promoting optimization method.

2. development of method that promotes problem-size reduction, benefitting from the recent development on signal processing and image reconstruction, that exploit sparsity structure of relevant information. In oil prospecting, due to the survey setting and limitation, seismic source and geophones are located sparsely that lead to data that are inherently sparse. Based on the theory of compressive sensing, information can still be reconstructed from far less measurements as needed by the classical Nyquist’s sampling theory.

Reference:


Prof.Francesco Sica (email: francesco.sica@nu.edu.kz)

Topic: Quantum-Resistant Cryptography Using Elliptic Curves
The need for security and encryption is particularly important in the digital age. Solutions have to be tested for efficiency and security against current technological advances. With the gradual emergence of quantum computers, a whole new security paradigm has to be investigated from little more than scratch. Such a preoccupation has also been reflected by the NSA already starting 2015, see www.iad.gov/iad/programs/iad-initiatives/cnsa-suite.cfm. In particular, Grover’s and especially Shor’s algorithms have shown that currently deployed public key cryptography will have to be completely overhauled with algorithms whose security is based on “quantum-resistant” computational problems, that is problems for which no significant speedup would be introduced in a quantum environment. One such problem is the isogeny problem for supersingular elliptic curves on which the security of the De Feo-Jao [1] key exchange algorithm is based. Many recent publications are aiming at analyzing the security of this recent work, as well as improving its efficiency. This is a current hot topic of research in elliptic curve cryptography. A PhD candidate will contribute to the study of both security and implementation of existing cryptosystems, with the possibility of approaching them from either the engineering side or the mathematical side.

References:

Prof. Manat Mustafa

Title: Computable structures and their algorithmic properties.

(Lead Supervisor: Manat Mustafa; Internal Co-Supervisor: Zhibek Kadyrsizova External Co-Supervisor: Nikolay Bazhenov)

During recent decades, usage of algorithms has become embedded in nearly every facet of our lives: science, industry, management, trade, medicine, etc. Advanced algorithms always rely on non-trivial mathematical tools. Therefore, this emphasizes the importance of the study of algorithmic properties for objects and structures from fundamental science and applications. The project will study algorithmic properties of mathematical structures with the help of modern methods of computable structure theory, recursion theory, and numbering theory. Computable structure theory is one of the traditional approaches to the study of algorithmic properties of mathematical objects. Many problems in this line of research are connected to the classification of algebraic structures by their level of complexity. Thus, one of the objectives of the project is the investigation of various features which express the complexity of algebraic
structures: their degree spectra, categoricity spectra, etc. The typical problems that arise here include the following questions: When does a mathematical structure have an algorithmic presentation? What does it mean to be algorithmically effective? How does one compare algorithmic presentations? What is a complex algorithmic presentation?
Here our focus is concentrated on algorithmic properties of several familiar classes of structures and to measure the algorithmic complexity of structures and classes.

Reference:
1. S. S. Goncharov, Countable Boolean Algebras and Decidability (Consultants Bureau, New York, 1997).

Title: Bell polynomials in differential algebra and determinant identities

Lead Supervisor: Manat Mustafa Internal Co-Supervisor: Durvudkhan Suragan and External Co-Supervisor: Askar Dzhumadil'daev

Bell polynomials appear in many places in combinatorics and physics. The aim of the project is to study matrices of Bell polynomials for differential operators.

Will be given an explicit construction of inverse map in Faa di Bruno Hopf algebra in terms of powers of differential operators. This construction will be used in establishing determinant identities whose components are differential polynomials.

References:
**Title: Cirquents: Logical foundations of mathematics and computation**

(Lead Supervisor: Prof. Manat Mustafa, manat.mustafa@nu.edu.kz, Internal Co-Supervisor: Prof. Ahti-Veikko Pietarinen ahtiveikko.pietarinen@nu.edu.kz, and External Co-Supervisor: Prof. Giorgi Japaridze,giorgi.japaridze@villanova.edu).

The aim of this project is to develop upon recent improvements of independent logics called cirquent calculus and to estimate their suitability as a foundational theory for mathematics and computability. Cirquent calculus is a proof system for constructs that are more general than formulas. Game-semantically conceived, it may become a new logic for computational problems. Most of the properties of fragments of cirquent calculus and its extensions and applications remain to be studied.

**References:**


**Prof. Durvudkhan Suragan**

1. **Functional inequalities and applications** (Lead Supervisor: Durvudkhan Suragan and External Co-Supervisor: Michael Ruzhansky).

We will deal with a fascinating area of mathematical analysis devoted to functional inequalities associated to operators with different geometries. This is an internationally very active area of research that witnessed a big boost during the last years. In this project we will aim at deriving new inequalities and at linking them to several problems of geometry and physics, as well as the analysis on groups and the calculus of variations.

**References:**


In this project we will deal with different models of partial differential equations with coefficients exhibiting singular behavior. It is well known that the classical theory of distributions does not apply in the case of strong singularities, however, recently new approaches have emerged based on the so-called very weak solutions. We will investigate properties of such solutions in several fundamental models from points of view of both pure and applied mathematics.

References:


In this project we will investigate the analytic objects appearing in the setting of non-Riemannian geometry by applying suitable versions of the Fourier analysis. We start with the analysis on compact manifolds (and compact Lie groups) where we introduce spaces related to sums of squares of vector fields. Such spaces can be well described in terms of their behavior on the
Fourier side which has to be adapted to the geometry of the space. The applications of such research will include new trends in harmonic analysis and new estimates for solutions of associated partial differential equations.

References:


4. Lie and Jordan elements in nonassociative algebras

Lead Supervisor: Durvudkhan Suragan Internal Co-Supervisor: Manat Mustafa and External Co-Supervisor: Askar Dzhumadil’daev

An element of free algebra is called Lie if it can be presented as a Lie polynomial on generators. Similarly, an element is Jordan if it is a Jordan polynomial on generators. Lie criterions for associative algebras are well known (Dynkin, Specht, Weber). Construction of Jordan criterion is an open problem even for the associative case. The aim of our project is to construct Lie and Jordan criteria for nonassociative algebras. It will be given algorithms for Zinbiel algebras to construct Lie and Jordan elements. These methods will be applied to establish analogs of Shirshov Cohn results for the specialty of Tortkara algebras.

References:
Over the past few years, neural networks have re-emerged as powerful machine learning (ML) models, yielding state-of-the-art results in fields such as computer vision, speech recognition, and natural language processing. One of the successful applications of feedforward neural networks is learning word embeddings, i.e. mapping words to vectors of real numbers, as epitomized by word2vec (Mikolov et al., 2013). Such embeddings form the bedrock of many natural language processing (NLP) tasks, from predictive language models and machine translation to image annotation and question answering, where they are largely ‘plugged in’ to a larger neural network architecture. An understanding of their properties is of interest as it may allow the development of better performing embeddings and improved interpretability of models using them. Tasks 1–3 below take steps in this direction, while Task 4 is a direct application of the results from Tasks 1–3 to a specific problem in Kazakh language processing.

1. Evaluate different methods of low rank approximation of matrices in the problem of obtaining PMI-based word embeddings. Levy and Goldberg (2014) showed that the SGNS model of Mikolov et al. (2013) is implicitly factorizing the pointwise mutual information (PMI) matrix shifted by a global constant. They also showed that the exact factorization with singular-value decomposition (SVD) can achieve solutions that are comparable to SGNS’s solutions. However, truncated SVD is not the only way of finding a low rank approximation of a matrix. It is optimal in the sense that it minimizes the approximation error in 2-norm, but this does not mean that it produces optimal word embeddings, which are usually evaluated in downstream tasks such as word similarity and word analogy. The question is: Is there any other method of low rank matrix approximation that produces word embeddings better than truncated SVD factorization? If no, what are the drawbacks of other methods compared to SVD?

2. Make the Rand-walk model of Arora et al. (2016) line up with the empirical observations from the common word embeddings methods. Recently a probabilistic model that explains the success of word embedding methods (CBOW, SGNS, Glove, etc.) was proposed by Arora et al. (2016). In their model, one of the key assumptions is that the so-called discourse vector should follow a random walk process over the unit sphere. If one looks carefully, the discourse vector in the work of Arora et al. (2016) mimics the behavior of the context embedding in the word2vec models but with one important distinction: context vectors in word2vec models (both CBOW and SGNS) are not restricted to have unit norm. Moreover, context vectors in word2vec are

Prof. Zhenisbek Assylbekov

Empirical evaluation and theoretical analysis of word embeddings with applications to Kazakh language processing
distributed similarly to word vectors in the sense that pairwise distances between input (word) embeddings strongly correlate with the corresponding pairwise distances between output (context) embeddings (Press and Wolf, 2017). The question is: Can we propose new assumptions for the Rand-walk model of Arora et al. (2016), which better fit these empirical observations? Will the main results of Arora et al. (2016) remain true under such assumptions?

3. Use the modified Rand-walk model to develop a method of tying weights in word2vec models. Tying input and output word embeddings is a regularization technique introduced earlier (Bengio et al., 2001; Mnih and Hinton, 2007) and studied in more details recently (Press and Wolf, 2017; Inan et al., 2017), which improves recurrent neural network language modeling (RNNLM) quality while decreasing the total number of trainable parameters almost two-fold, since most of the parameters in RNNLM are due to embedding matrices. Unfortunately, direct application of this regularization technique to the SGNS model worsens the quality of word vectors (Press and Wolf, 2017), which was predicted (but not verified experimentally) by Goldberg and Levy (2014). We agree with the argument of Goldberg and Levy (2014) that trivial reusing of input embeddings at output is not appropriate for SGNS, but we believe that there exists a non-trivial way to do that. We hypothesize that the relationship between input and output embeddings in word2vec models is linear but not identity. There is an empirical evidence supporting this hypothesis (Mimno and Thompson, 2017) and we would like to develop a theoretical justification of it: we will use the Rand-walk model with the modified assumptions (from Task 2) to develop a theoretically-driven method of reusing input embeddings at output in the word2vec models.

4. Apply word embeddings to automate the process of cleaning and extending the existing morphological analyzer for Kazakh. Currently the PI of this proposal runs a separate project (funded by the Ministry of Education and Science of the Republic of Kazakhstan, contract #346/018-2018/33-28, IRN AP05133700), where one of the tasks is cleaning and extending the existing free/open-source morphological analyzer for Kazakh apertium-kaz (Washington et al., 2014). This is done manually by a team of annotators. At the time of this writing, the transducer has 35,921 stems in its lexicon and covers 94.5% of a running text. Since we have a significant amount of labeled (but noisy) data in the lexicon (stems and tags), and we can extract “semantics” for each word from large amount of unannotated data through word2vec, we hypothesize that there is a way to train a classifier, which can ease the job of human annotators when adding new stems to the lexicon and removing irrelevant ones.

References


**Prof. Vasileios Zarikas** vasileios.zarikas@nu.edu.kz

### 1. Bayesian Networks for engineering and science applications

The only mathematically consistent methodology to drive rational decisions is the framework of Bayesian reasoning. New techniques for the problem of assigning probabilities to large Conditional Probability Tables (CPT) will be developed. Automatic ways to fill the CPTs from fuzzy rules or incomplete databases will be structured [1]. Furthermore, the utilization of Dynamical Bayesian networks in various engineering decision problems will be explored. Special focus will be given to apply
Bayesian reasoning in renewable energy applications such as wind energy, photovoltaic energy or smart grids[2]. Requirements: mathematical and programming skills in C


2. Mathematical models of modified gravities
In recent years, the cosmological puzzles of dark matter, dark energy and coincidence problem directed research towards the further development of the so-called modified gravities and renormalization group (RG) approaches to quantum gravity [1],[2],[3],[4] The program will study phenomenological aspects of some of these new theories and it may propose modifications to some of these new alternative models of gravity. The method of matching spacetime solutions will be improved and utilized for the case of quantum-corrected black holes and cosmological solutions. Requirements: Strong mathematical skills


3. Regulatory framework for Artificial Intelligence (AI)
Last years, there is a debate about how to regulate and in what extend AI, [1,2,3]. A state regulatory framework seems necessary to avoid harm. However, regulations and laws are also a slow-moving tool
which may create several problems in the development of technology. Furthermore, regulation experiences political interference and public unreasonable fear.

AI is a very fast-moving area of research and unwise regulations may set a barrier to innovation and future developments. AI can propose important solutions to vehicle safety, improved productivity, health system etc. Nevertheless, AI experts forecast also potential significant risks when true AI systems will be developed capable to develop new non trivial knowledge (mathematical theorems, patents etc.). Elon Musk has urged U.S. governors to regulate AI “before it’s too late”. AI systems will eventually overtake the ability of humans to understand future Science. The latter means that humans will not be able anymore to follow the reasoning of an AI system decisions, assuming the best case that AI will provide explanations (which is not always true i.e. Neural networks).

The regulatory framework will be developed in a mathematical consistent way using Bayesian networks and relevant mathematical utilities that will represent the involved ethical policies and the rival importance of the proposed AI solution.

